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Name of Examination : **Winter 2020** - (Preview)

Course Code & Course Name : **SH101U - Differential Calculus**

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Maximum Marks : **60**

Duration : **3 Hrs**

[Edit](#) [Print](#) [View Answer Key](#) [Close](#) **Answer Key Submission Type:** Marking scheme with model answers and solutions of numerical

Instructions:

1. All questions are compulsory.
2. Illustrate your answer with suitable figures/sketches wherever necessary.
3. Assume suitable additional data; if required.
4. Use of logarithmic table, drawing instruments and non programmable calculators is allowed.
5. Figures to the right indicate full marks.

1) Solve all the questions

a) Find Eigen values and Eigen vectors of the following matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ [5]

b) Find the non singular matrices P & Q such that PAQ is in normal form. [5]

Hence find the rank of A . where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$

c) Given the transformation $Y = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ [4]

find the coordinates (x_1, x_2, x_3) in X corresponding to $(1, 2, -1)$ in Y

2) Solve any three questions

a) If $x = 2 \cos \theta \cos h\phi$, $y = 2 \sin \theta \sin h\phi$, Prove that $\sec(\theta + i\phi) - \sec(\theta - i\phi) = \frac{4iy}{x^2 + y^2}$ [4]

b) Separate into real & imaginary parts of the expression i^i [4]

c) Find the cube root of $(1 - \cos \theta - i \sin \theta)$ [4]

d) Prove that $\sin^{-1}(\cos e\theta) = \frac{\pi}{2} + i \log \cot\left(\frac{\theta}{2}\right)$ [4]

3) Solve any three questions

a) If $y = \left[\log \left(x + \sqrt{1+x^2} \right) \right]^2$ then Prove that $(1+x^2) y_{n+2} + (2n+1) x y_{n+1} + n^2 y_n = 0$ [4]

b) Expand $\sin^{-1} x$ in ascending powers of x [4]

c) Using Taylor's theorem , expand $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$ [4]

d) Evaluate $\lim_{x \rightarrow 0} \left[\frac{a^x + b^x}{2} \right]^{1/x}$ [4]

4) Solve any three questions

a) Find the equation of the right circular cylinder of radius 2 whose axis passes through $(1, 2, 3)$ and has direction cosine proportional to 2,-3,6. [4]

b) Obtain the equation of the right circular cone which passes through the point $(2, 1, 3)$ with vertex at $(1, 1, 2)$ and axis parallel to the line $\frac{x-2}{2} = \frac{y-1}{-4} = \frac{z+2}{3}$ [4]

c) Prove that the plane $2x - 2y + z + 12 = 0$ touches the sphere $(x^2 + y^2 + z^2) - 2x - 4y + 2z - 3 = 0$ and find the point of contact. [4]

d) Examine for L.D. or L.I. of vectors $(2, -1, 3, 2)$, $(1, 3, 4, 2)$, $(3, -5, 2, 2)$ Find the relation between them , if dependent. [4]

5) Solve any two questions

a) Use Lagrange's method to find the minimum distance from origin to the plane $3x + 2y + z = 12$. [5]

b) If $f(lx + my + nz, x^2 + y^2 + z^2) = 0$, Prove that $(lz - nx) + (mz - ny) \frac{\partial y}{\partial x} + (ly - mx) \frac{\partial y}{\partial z} = 0$ [5]

c) Examine for minimum and maximum values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ [5]

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